Test 3 / Numerical Mathematics 1 / June 18th 2021, University of Groningen

Instructions

- The test has two pages.
- This examination is *open book*, that is you are allowed to check any material of the course.
- **You have 1 hour and 45 minutes in total to finalize the test and upload your solutions. People with special needs (according to the official information of the Educational support center) have 2 hours in total.**
- Upload the answers on the same location in Nestor where you downloaded them, not in your File Exchange.
- The grade will be computed as the number of obtained points, plus 1.
- Do not communicate with other students during the examination.
- If you did not write Test 1 and hence you have not yet upload the pledge, please sign the pledge and upload it as a separate file in the same place where you will upload your answers.
- Stay in the online main collaborate room during the examination. Information delivered there is official and part of the instructions of the examination. The activity in the room will be recorded.
- The examination must be written by hand **in a tidy and legible way**, scanned and uploaded as PDF to Nestor. Of course you can also use a tablet to write your answers.
- **Upload the PDF in vertical orientation, such that it requires no rotation to be readable.**
- All answers need to be justified using mathematical arguments.
- Oral checks may be run afterwards, either randomly and/or in case of suspicion of fraud.
- **If you do not follow these instructions you will receive the minimal grade.**

Questions

Consider the ODE for the function $z(t)$: $z' = f(z)z, z(0) = z_0 \neq 0$ with $f(z) < 0 \ \forall z$.

(a) $\boxed{1.0}$ Show that $(z^2)' < 0$.

(b) $\boxed{1.0}$ Show that for the numerical method for the ODE given by: $z_{n+1} - z_n = hf(z_n)z_{n+1}$, where $z_n \approx z(t_n)$ and $h = t_{n+1} - t_n > 0$, it holds that, $(z_{n+1})^2 < (z_n)^2$ for $z_0 \neq 0$.

(c) $\boxed{1.0}$ Formulate Newton iterations to find z_{n+1} and perform two iterations by hand.

Consider the system of ODEs for the functions $y(t), x(t)$:

$$\begin{aligned} y' &= -\alpha y^p + x^q y^m \ , y(0) = y_0 \neq 0 \\ x' &= -x^{q-1} y^{m+1} - \gamma x^r \ , x(0) = x_0 \neq 0 \end{aligned} \tag{1}$$

with $\alpha, \gamma > 0$ and $p, q, m, r \in \mathbb{N}$.

(d) $\boxed{1.5}$ Show that for p, r odd numbers, it holds:

$$\frac{1}{2}(y^2 + x^2)' < 0$$

(e) 1.0 Formulate a root-finding problem for $x_{n+1} \approx x(t_{n+1})$ and $y_{n+1} \approx y(t_{n+1})$ by applying the β-method to (1).

(f) 1.0 Compute the Jacobian of the residual from previous question.

(g) 1.5 Show that for p, r odd numbers, $x_{n+1}^2 + y_{n+1}^2 < x_n^2 + y_n^2$ for $\beta \geq 1/2$. You may need to use the identity:

$$(z_{n+1} - z_n)z_{n+\beta} = z_{n+1}^2/2 - z_n^2/2 + (\beta - 1/2)(z_{n+1} - z_n)^2$$

(h) 1.0 What would be the most convenient choice of β for performing computations of the $\beta-$method in Question (e) question. Justify your choice. Please give also one disadvantage of your choice.